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**Chapter # 05 (Exponential and Logarithmic Function)**

**5.5 Properties of Logarithms:**

**Objectives:** 1 Work with the Properties of Logarithms

2 Write a Logarithmic Expression as a Sum or Difference of Logarithms

3 Write a Logarithmic Expression as a Single Logarithm

4 Evaluate Logarithms Whose Base Is Neither 10 Nor *e*

Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents.

**Example1:** (a) Show that l1=0, (b) Show that .

**Solution:** **(a)** Let,

Then,

proved.

**(b)** Let,

Then,

proved.

**Theorem: Properties of Logarithms**

In the following properties *M*, *N*, and *a* are positive real numbers, *a* ≠ *1*, and *r* is any real number.

**(i)** The Log of a Product Equals the Sum of the Logs:

**(ii)** The Log of a Quotient Equals the Difference of the Logs:

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**(iii)** The Log of a Power Equals the Product of the Power and the Log:

**Proof: (i)** We have to show that,

Where, *M*, *N*, and *a* are positive real numbers, *a* ≠ *1*, and *r* is any real number.

Let, and

and

Now,

i.e. proved

**(ii)** We have to show that,

Where, *M*, *N*, and *a* are positive real numbers, *a* ≠ *1*, and *r* is any real number.

Let, and

and

Now,

i.e. proved.

**Example3:** Write, , as a sum of logarithms. Express all power as factors.

**Solution:**

Ans.

**Example5:** Write, , as a sum and difference of logarithms. Express all powers as factors.

**Solution:**

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Ans.

**Theorem:** Properties of Logarithms In the following properties *M*, *N*, and *a* are positive real numbers,

**(i)** If , then

**(ii)** If , then

**Theorem (Change-of-base Formula):** If and *M* are positive real numbers, then

**Proof:** Let, , then

proved.

**Home Work: Exercise 5.5: Problem No. 13 - 78 and 87 - 103**

**Exercise 5.5:**

**Question no. 13 - 28 are same:**

**Question 19:** Find the exact value of,

**Solution:** Ans.

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**Question 23:** Find the exact value of,

**Solution:** (use change of base rule)

Ans.

**Question no. 29 - 36 are same:**

**Question 35:** Suppose that and . Use properties of logarithms to write following logarithm in terms of *a* and *b*: .

**Solution:** Ans.

**Question no. 37 - 56 are same:**

**Question 49:** Write the following expression as a sum and/or difference of logarithms. Express powers as factors:

**Solution:**

Ans.

**Question no. 57 - 70 are same:**

**Question 61:** Write the following expression as a single logarithm:

**Solution:** Given,

Ans.

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**Question no. 71 - 78 are same:**

**Question 74:** Use the Change-of-Base Formula and a calculator to evaluate following logarithm. Round your answer to three decimal places: .

**Solution:** Given, Ans.

**Question no. 87 - 103 are same:**

**Question 94:** Express y as a function of x for the following expression, where the constant C is a positive number:

**Solution:**

Ans.

**Question 101:** Show that,

**Proof:** Left Hand Side

Proved.